

Tezukayama RIEB Discussion Paper Series No. 13

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competitive model with the removal of zero-profit conditions

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# The equilibrium distribution of firms in a monopolistically competitive model with the removal of zero-profit conditions

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## Abstract

In most existing open-economy monopolistically competitive models, zero-profit conditions are indispensable for determining the number of firms in each country. In our model, we remove the zero-profit conditions from the standard monopolistically competitive settings. Instead, we construct a monopolistically competitive model with a linear segment for the distribution of firms. In such a model, we can show that the equilibrium distribution of firms always exists even with the removal of the zero-profit conditions. In addition, we provide the parameter condition under which each firm always has a strictly positive pure profit. The condition shows that when one country's fixed cost is small enough for a given level of the other country's fixed cost, the equilibrium pure profits are strictly positive. As in the literature, we also find that the larger the country size, the larger the share of firms in that country. In addition, under asymmetric fixed costs, we find that a unilateral increase in one country's fixed cost decreases the share of firms in that country.

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## I. Introduction

In most existing two-country monopolistically competitive models, fixed costs and zero-profit conditions are indispensable for determining the number of firms in each country (the seminal contribution is Krugman (1979, 1981)). Generally, the presence of fixed costs serves two purposes. On the one hand, it increases the incentive for a firm to concentrate production in only one location (economies of scale). On the other hand, it guarantees that free entry of firms drives pure profits to zero, thereby determining the number of firms in each country (zero-profit condition). Although the mathematical tractability of monopolistically competitive models is improved by the zero-profit conditions, it appears to be at odds with reality because in the real world most firms earn positive pure profits. Hence, the use of a monopolistic competition model without zero-profit conditions (or with strictly positive pure profits) may be more realistic for economic analysis.

The purpose of this paper is to show the existence of the equilibrium distribution of firms (or each country's share of firms) within a monopolistically competitive framework even with the removal of the zero-profit conditions.

There are some crucial differences between our model and that of Krugman (1979, 1981). The most important point to note is that our paper assumes a linear segment for the distribution of firms,  $n \in [0, 1]$ , where firms in the interval  $[0, n]$  are located in the home country, and the remaining  $(n, 1]$  firms are located in the foreign country. Therefore,  $n(1 - n)$  measures the home (foreign) country's share of firms. We assume the linear segment for tractability so that we can solve the model analytically for the distribution of firms even with the removal of the zero-profit conditions.

A considerable number of recent theoretical studies have examined the international distribution of firms based on economies of scale and zero-profit conditions. In the new trade theories literature, Krugman (1979, 1980, 1981) and Helpman and Krugman (1985) show the role of economies of scale as a determinant of the patterns of trade.<sup>1</sup> In their models, however, because zero-profit conditions are required to determine the number of firms in each country, the pure profit of each firm becomes zero in

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<sup>1</sup> Other contributions to this literature include Venables (1987), Kikuchi (2004), Kikuchi and Zeng (2004), Laussel and Paul (2007), and Melitz and Ottaviano (2008), among others.

equilibrium through free entry. Similarly, in the multinational firm literature, zero-profit conditions are also required to determine the number of multinational firms (e.g. Ethier and Markusen, 1996; Markusen and Venables, 1998, 1999; Helpman *et al.*, 2004; Eckel and Egger, 2009). The third strand concerns the new economic geography (NEG) literature on mobile capital and endogenous growth. This literature includes Martin and Rogers (1995), Martin (1999), Martin and Ottaviano (1999, 2001) and Johdo (2013). The characteristic feature of the NEG models is that a patent is required to begin producing each variety of good, and those authors interpret this capital requirement as a fixed production cost.<sup>2</sup> Because of the presence of such a fixed cost, in the NEG models, zero-profit conditions are also needed to determine the international distribution of firms (or mobile capital).

This paper will show that the zero-profit conditions are not essential for the existence of the equilibrium distribution of firms within a monopolistically competitive framework.

The remainder of this paper is structured as follows. Section II outlines the features of the model and describes the equilibrium international distribution of firms. Then, we provide parameter conditions under which the firms earn a strictly positive pure profit. Next, we examine the impacts of a changing relative market size on the equilibrium share of firms. Finally, we examine how the asymmetry of fixed costs affects the international distribution of firms. The final section summarizes the findings and concludes the paper.

## **II. The Model**

We assume a two-country world economy, with a home country and a foreign country. The models for the home and foreign countries are the same; an asterisk is used to denote foreign variables. There are only horizontally differentiated goods, which are assumed to be traded freely. The differentiated goods are subject to a monopolistically competitive market structure. In addition, the differentiated goods are assumed to be produced using an increasing-returns technology that requires labor as the only input.

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<sup>2</sup> In addition, in order to exclude the case where the patent can be used in more than one location at once, they need to interpret the patent as a fixed cost inclusive of a piece of machinery.

The market for labor is perfectly competitive. Monopolistically competitive firms exist continuously in the world in the  $[0, 1]$  range, where each firm produces a differentiated product in only one location because of increasing returns-to-scale technology. Monopolistically competitive firms are mobile across countries, but their owners are not. Hence, all profit flows are distributed to the immobile owners. In addition, firms in the interval  $[0, n]$  are located in the home country, and the remaining  $(n, 1]$  firms are located in the foreign country, where  $n$  is endogenous. Therefore,  $n(1-n)$  measures the home (foreign) country's share of firms. The size of the world population is normalized to unity and therefore  $s + s^* = 1$ , where  $s$  ( $s^*$ ) measures the relative size of the home (foreign) country's population.

Preferences are defined over differentiated goods, named  $C$ . In this paper, the preferences of household  $i \in (0, s)$  in the home country are represented by the following utility function:<sup>3</sup>

$$U^i = u(C^i), \quad (1)$$

where  $u'(C^i) > 0$  and  $u''(C^i) < 0$ . In equation (1), the consumption index  $C^i$  is defined as follows:

$$C^i = \left( \int_0^n C^i(j)^{(\theta-1)/\theta} dj + \int_n^1 C^{i*}(j)^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)}, \quad (2)$$

where  $\theta > 1$  measures the elasticity of substitution between any two differentiated goods and  $C^i(j)$  is the consumption of good  $j$  for household  $i$ . The consumption price index is defined as:

$$P = \left( \int_0^n P(j)^{1-\theta} dj + \int_n^1 (P^*(j))^{1-\theta} dj \right)^{1/(1-\theta)}, \quad (3)$$

where  $P(j)$  is the price of good  $j$  produced in the home country. The value of expenditure for household  $i$ ,  $E^i$ , is defined as follows:

$$E^i = \int_0^n P(j)C^i(j)dj + \int_n^1 P^*(j)C^i(j)dj. \quad (4)$$

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<sup>3</sup> In what follows, we focus mainly on the description of the home country because the foreign country is described analogously.

We assume that every household supplies one unit of labor to domestic firms at the domestic wage rate. We also assume that every household holds the stocks of all firms with the same portfolio. Therefore, they receive profits through dividends equally from all firms. Then, the household budget constraint can be written as:

$$E^i = W + \int_0^n \Pi(j) dj + \int_n^1 \Pi^*(j) dj, \quad (5)$$

where  $W$  denotes the nominal wage rate and  $\Pi(j)$  ( $\Pi^*(j)$ ) is the nominal profit flow of firm  $j$  located at home (abroad).<sup>4</sup> In other words,  $\Pi(j)$  ( $\Pi^*(j)$ ) represents the nominal flow of dividends from holding equities of firm  $j$  located at home (abroad).

Households in the home (foreign) country maximize (2) subject to (4) by allocating  $C^i(j)$  optimally. This problem yields:

$$C^i(h) = \left( \frac{P(h)}{P} \right)^{-\theta} \left( \frac{\alpha E^i}{P} \right), \quad (6a)$$

$$C^i(f) = \left( \frac{P^*(f)}{P} \right)^{-\theta} \left( \frac{\alpha E^i}{P} \right).^5 \quad (6b)$$

The households are supposed to be symmetric, so we can delete the superscript  $i$  from  $E^i$ . Aggregating the demands in (6a), (6b) and those of foreign counterparts across all households worldwide yields the following market clearing condition for any differentiated product  $h$ ,  $x(h)$ :

$$\begin{aligned} x(h) &= s \left( \frac{P(h)}{P} \right)^{-\theta} \left( \frac{E}{P} \right) + s^* \left( \frac{P(h)}{P^*} \right)^{-\theta} \left( \frac{E^*}{P^*} \right) \\ &= \left( \frac{P(h)}{P} \right)^{-\theta} e^w, \end{aligned} \quad (7)$$

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<sup>4</sup> In a related two-country monopolistic trade model, Corsetti *et al.* (2007) also posit that each home and foreign household receives an equal share of profits of all firms as shown in (5).

<sup>5</sup> We have used the index  $h$  to denote the symmetric values within the home country, and we have used the index  $f$  for the foreign country.

where the second line of (7) is derived by substituting  $P(h)/P = P(h)/P^*$  into the first line, taking account of the aggregate per capita world consumption  $e^w \equiv s(E/P) + s^*(E^*/P^*)$ . Similarly, for any product  $f$  of the foreign located firms, we obtain:

$$\begin{aligned} x(f)^* &= s\left(\frac{P^*(f)}{P}\right)^{-\theta} \left(\frac{E}{P}\right) + s^*\left(\frac{P^*(f)}{P^*}\right)^{-\theta} \left(\frac{E^*}{P^*}\right) \\ &= \left(\frac{P^*(f)}{P^*}\right)^{-\theta} e^w. \end{aligned} \quad (8)$$

In the monopolistic goods sector,  $x(h) + F$  units of labor are required to produce  $x(h)$  units of a variety where  $F$  is the fixed cost. Because home-located firm  $h$  hires labor domestically, given  $W, P, E, E^*$ , and  $n$ , and subject to (7), home-located firm  $h$  faces the following profit-maximization problem:

$$\max_{P(h)} \Pi(h) = (P(h) - W)x(h) - WF. \quad (9)$$

By substituting  $x(h)$  from (7) into the firm's nominal profit  $\Pi(h)$  and then differentiating the resulting equation with respect to  $P(h)$ , we obtain the following price markup:

$$P(h) = \left(\frac{\theta}{\theta - 1}\right)W. \quad (10)$$

Substituting (7) and (10) and those of the foreign counterparts into the real profit flows of the home- and foreign-located firms,  $\Pi(h)/P$  and  $\Pi(f)^*/P^*$ , respectively, we obtain:

$$\frac{\Pi(h)}{P} = \left(\frac{1}{\theta}\right)\left(\frac{P(h)}{P}\right)^{1-\theta} e^w - \left(\frac{\theta - 1}{\theta}\right)\left(\frac{P(h)}{P}\right)F, \quad (11a)$$

$$\frac{\Pi(f)^*}{P^*} = \left(\frac{1}{\theta}\right)\left(\frac{P(f)^*}{P^*}\right)^{1-\theta} e^w - \left(\frac{\theta - 1}{\theta}\right)\left(\frac{P(f)^*}{P^*}\right)F^*. \quad (11b)$$

Here, substituting  $P(j) = P(h)$  and  $P^*(j) = P^*(f)$  into (3) and that of the foreign counterparts, respectively, we obtain the following real prices:

$$P(h)/P = P(h)/P^* = \omega^{-1} [n\omega^{\theta-1} + (1-n)]^{-1/(1-\theta)}, \quad (12a)$$

$$P^*(f)/P = P^*(f)/P^* = [n\omega^{\theta-1} + (1-n)]^{-1/(1-\theta)}, \quad (12b)$$

where  $\omega \equiv P(f)/P(h)$  is the price of foreign goods relative to home goods. The home labor-market clearing condition reduces to  $s = n(x(h) + F)$ . Substituting (7) into this, and using (12a), we obtain:

$$s = n\omega^\theta [n\omega^{\theta-1} + (1-n)]^{\theta/(1-\theta)} e^w + nF. \quad (13)$$

Similarly, from the foreign labor-market clearing condition, we obtain:

$$s^* = (1-n) [n\omega^{\theta-1} + (1-n)]^{\theta/(1-\theta)} e^w + (1-n)F^*. \quad (14)$$

The model assumes that firms do not face any relocation costs. For a firm to be indifferent between home and foreign locations after location arbitrage, returns from the two locations must be equalized:

$$\Pi(h)/P = \Pi(f)^*/P^*. \quad (15)$$

This condition guarantees that some firms exit in both countries. By substituting (11a) and (11b) into (15), and using (12a), (12b), (13), and (14), we obtain the equilibrium distribution of firms that satisfies the following expression:<sup>6</sup>

$$\begin{aligned} & n^{(1-\theta)/\theta} (s - nF)^{(\theta-1)/\theta} - (\theta - 1)n^{1/\theta} (s - nF)^{-1/\theta} F \\ & = (1-n)^{(1-\theta)/\theta} (s^* - (1-n)F^*)^{(\theta-1)/\theta} - (\theta - 1)(1-n)^{1/\theta} (s^* - (1-n)F^*)^{-1/\theta} F^*. \end{aligned} \quad (16)$$

Equation (16) shows that the left-hand side depends negatively on  $n$  monotonically and the right-hand side depends positively on  $n$  monotonically. Therefore, as seen in Figure 1, the equilibrium distribution of firms ( $= n_e$ ) always exists uniquely.

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<sup>6</sup> See the Appendix for the derivation of equation (16).



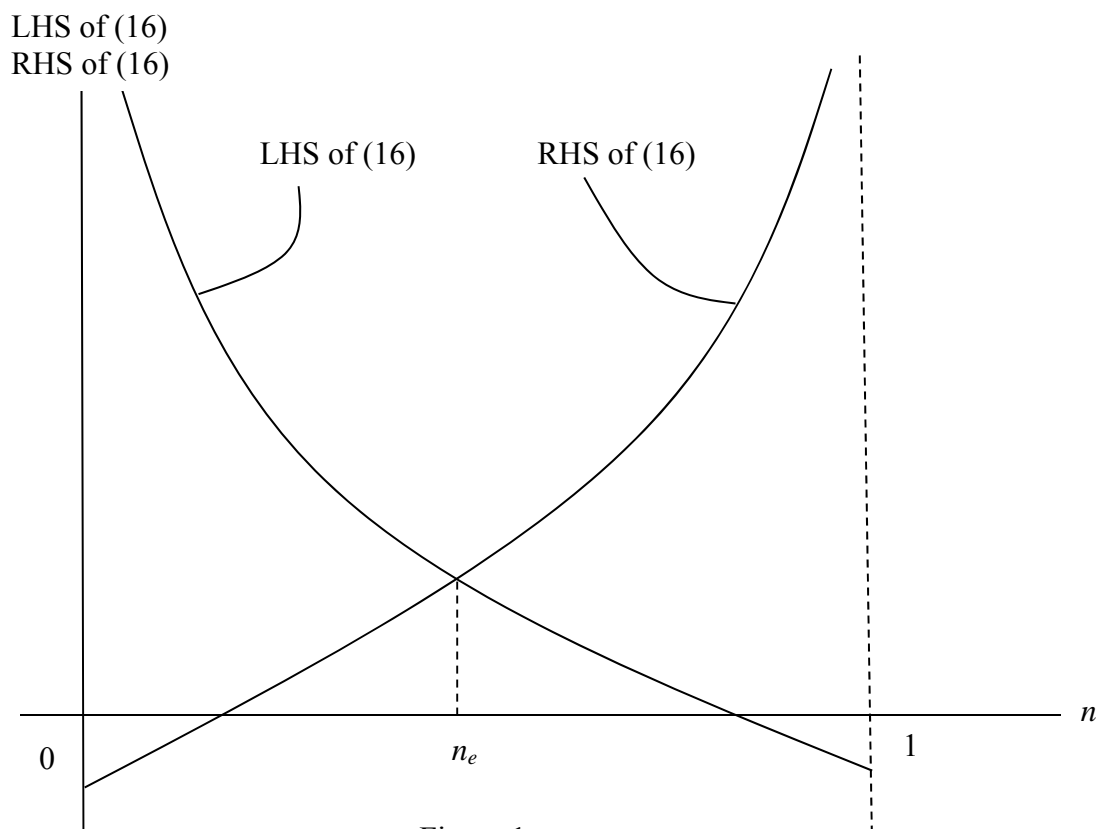


Figure 1

Let us now turn to the question of stability. As usual, the interior equilibrium of the distribution of firms ( $n_e \in (0, 1)$ ) is stable only if  $\partial(\Pi(h)/P - \Pi^*(f)/P^*)/\partial n < 0$  because in this case, a positive shock to  $n$  decreases  $\Pi(h)/P$ , increases  $\Pi^*(f)/P^*$  and hence generates relocation of firms that corrects the initial perturbation.<sup>7</sup> Therefore, to answer the question of stability, we only need to look at the sign of  $\partial(\Pi(h)/P - \Pi^*(f)/P^*)/\partial n$  at  $n_e \in (0, 1)$ .<sup>8</sup> Using equations (15) and (16) to form  $\Pi(h)/P - \Pi^*(f)/P^*$  and differentiating with respect to  $n$ , we have<sup>9</sup>

<sup>7</sup> See, for example, Baldwin *et al.* (2003) for a stability analysis in the NEG models.

<sup>8</sup> From equations (15) and (16), we find that the real profit differential  $\Pi(h)/P - \Pi^*(f)/P^*$  is a continuous function of  $n$ .

<sup>9</sup> In equation (16), the left-hand side represents  $\Pi(h)/P$  and the right-hand side represents  $\Pi^*(f)/P^*$ .

$$\begin{aligned} \frac{\partial(\Pi(h)/P - \Pi^*(f)/P^*)}{\partial n} = & -\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{n_e} + \frac{F}{s-n_e F}\right)\left[\left(\frac{n_e}{s-n_e F}\right)^{(1-\theta)/\theta} + \left(\frac{n_e}{s-n_e F}\right)^{1/\theta} F\right] \\ & -\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{1-n_e} + \frac{F^*}{s^*-(1-n_e)F^*}\right)\left[\left(\frac{1-n_e}{s^*-(1-n_e)F^*}\right)^{(1-\theta)/\theta} + \left(\frac{1-n_e}{s^*-(1-n_e)F^*}\right)^{1/\theta} F^*\right] < 0. \end{aligned}$$

Thus, in our model, the interior equilibrium is always stable. Moreover, from equations (15) and (16), we find the parametric condition required for the equilibrium profits to be strictly positive (i.e.,  $\Pi(h)/P = \Pi(f)^*/P^* > 0$ ):

$$\frac{s}{\theta F} + \frac{s^*}{\theta F^*} > 1. \quad (17)$$

Therefore, from (17), if one country's fixed cost is small enough for a given level of the other country's fixed cost, all firms always earn a strictly positive pure profit. In addition, given the equilibrium distribution of firms, from (13) and (14), the equilibrium relative price,  $\omega_e \equiv P(f)_e/P(h)_e$ , and aggregate per capita world consumption,  $e_e^w$ , are, respectively:

$$\omega_e = \left(\frac{1-n_e}{n_e}\right)^{\frac{1}{\theta}} \left(\frac{s-n_e F}{s^*-(1-n_e)F^*}\right)^{\frac{1}{\theta}}, \quad (18)$$

$$e_e^w = (1-n_e)^{-1} \left[ n_e \omega_e^{\theta-1} + (1-n_e) \right]^{\frac{\theta}{1-\theta}} \left[ s^* - (1-n_e)F^* \right]. \quad (19)$$

Accordingly, from (11a) and (11b), we obtain the equilibrium real profit flows of the home- and foreign-located firms,  $\Pi(h)_e/P_e$  and  $\Pi(f)_e^*/P_e^*$ . Moreover, from (4), (6a), and (6b), we obtain the real values of expenditure for home and foreign households,  $E_e/P_e$  and  $E_e^*/P_e^*$ . Finally, from (1) and considering  $C_e = E_e/P_e$  and  $C_e^* = E_e^*/P_e^*$ , we obtain the equilibrium utility in each country,  $u(C_e)$  and  $u^*(C_e^*)$ .

We now examine the impacts of a changing relative market size on the equilibrium share of firms. Differentiating (16) with respect to the relative home-market size,  $s$ , we obtain:

$$\frac{dn}{ds} = \frac{n^{\frac{1}{\theta}-1} s (s - nF)^{-\left(\frac{1+\theta}{\theta}\right)} + (1-n)^{\frac{1}{\theta}-1} s^* [s^* - (1-n)F^*]^{-\left(\frac{1+\theta}{\theta}\right)}}{n^{\frac{1}{\theta}-2} s^2 (s - nF)^{-\left(\frac{1+\theta}{\theta}\right)} + (1-n)^{\frac{1}{\theta}-2} (s^*)^2 [s^* - (1-n)F^*]^{-\left(\frac{1+\theta}{\theta}\right)}} > 0. \quad (20)$$

The result in (20) shows that an increase in  $s$  leads to the relocation of some firms away from the foreign country to the home country ( $dn/ds > 0$ ). This is because an increase in  $s$  leads to an increase in the market size of the home country, and therefore this increases the relative profit of firms located in the home country.

In our model, we have assumed asymmetric fixed costs (i.e.,  $F \neq F^*$ ).<sup>10</sup> Therefore, we can examine how the asymmetry of fixed costs affects the international distribution of firms. From (16), the impact of a rise in the home country's fixed cost  $F$  on the distribution of firms is:

$$\frac{dn}{dF} = -\frac{n^{\frac{1}{\theta}} (s - nF)^{\frac{1}{\theta}} \left[ \frac{s}{s - nF} + \theta \right]}{n^{\frac{1}{\theta}-2} s^2 (s - nF)^{-\left(\frac{1+\theta}{\theta}\right)} + (1-n)^{\frac{1}{\theta}-2} (s^*)^2 [s^* - (1-n)F^*]^{-\left(\frac{1+\theta}{\theta}\right)}} < 0. \quad (21)$$

The result in (21) shows that an increase in  $F$  leads to a relocation of some firms away from the home country to the foreign country ( $dn/dF < 0$ ). This is because an increase in  $F$  leads to a decrease in the relative profit of firms located in the home country, and therefore this leads to the relocation of some firms away from the home country to the foreign country.<sup>11</sup>

In contrast, the impact of a rise in the foreign country's fixed cost,  $F^*$ , on the distribution of firms is:

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<sup>10</sup> In contrast, the new trade theory literature often assumes symmetric fixed costs. But there are a few exceptions in the literature; for example, Kikuchi (2004) and Kikuchi and Zeng (2004). They generalize the model of Krugman (1979) to incorporate cross-country production cost differences and explore the role of the asymmetry of production costs as a determinant of the patterns of trade.

<sup>11</sup> In the open economy macroeconomics literature, Ghironi and Melitz (2005) and Corsetti *et al.* (2007) show that a real shock to entry costs alters the international distribution of firms. In this literature, however, zero-profit conditions are required to determine the number of firms in each country.

$$\frac{dn}{dF^*} = \frac{(1-n)^{\frac{1}{\theta}} [s^* - (1-n)F^*]^{\frac{1}{\theta}} \left[ \frac{s^*}{s^* - (1-n)F^*} + \theta \right]}{n^{\frac{1}{\theta}-2} s^2 (s-nF)^{-\left(\frac{1+\theta}{\theta}\right)} + (1-n)^{\frac{1}{\theta}-2} (s^*)^2 [s^* - (1-n)F^*]^{\left(\frac{1+\theta}{\theta}\right)}} > 0. \quad (22)$$

The result in (22) shows that an increase in  $F^*$  leads to a relocation of some firms away from the foreign country to the home country ( $dn/dF^* < 0$ ).<sup>12</sup>

### III. Concluding Remarks

This paper analyzed whether or not the open-economy monopolistically competitive model with the removal of the zero-profit conditions can provide the equilibrium distribution of firms. The main findings of our analysis are that: 1) even with the removal of the zero-profit conditions, the equilibrium number of firms in each country can be derived endogenously within a monopolistically competitive framework, and 2) when one country's fixed cost is small enough for a given level of the other country's fixed cost, the equilibrium profits are strictly positive. We also find that the larger the country size, the larger the share of firms in that country. In addition, we find that an increase in the home (foreign) country's fixed cost leads to a relocation of some firms away from the home (foreign) country to the foreign (home) country.

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<sup>12</sup> In most of the existing literature on new trade theories and the NEG, the relative number of firms in each country is independent of the fixed costs. This is because the fixed costs of individual countries are identical between countries, and hence, changes in the fixed costs of both countries offset each other.

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## Appendix

Substituting (11a) and (11b) into (15) yields:

$$\left(\frac{1}{\theta}\right)\left(\frac{P(h)}{P}\right)^{1-\theta} e^w - \left(\frac{\theta-1}{\theta}\right)\left(\frac{P(h)}{P}\right)F = \left(\frac{1}{\theta}\right)\left(\frac{P(f)^*}{P^*}\right)^{1-\theta} e^w - \left(\frac{\theta-1}{\theta}\right)\left(\frac{P(f)^*}{P^*}\right)F^*. \quad (\text{A.1})$$

Substituting (12a) and (12b) into (A.1) yields:

$$\begin{aligned} & \left(\frac{1}{\theta}\right)\omega^{\theta-1}[n\omega^{\theta-1} + (1-n)]^{-1} e^w - \left(\frac{\theta-1}{\theta}\right)\omega^{-1}[n\omega^{\theta-1} + (1-n)]^{-1/(1-\theta)} F \\ & = \left(\frac{1}{\theta}\right)[n\omega^{\theta-1} + (1-n)]^{-1} e^w - \left(\frac{\theta-1}{\theta}\right)[n\omega^{\theta-1} + (1-n)]^{-1/(1-\theta)} F^*. \end{aligned} \quad (\text{A.2})$$

Equation (13) can be written as:

$$\omega^{\theta-1}[n\omega^{\theta-1} + (1-n)]^{-1} = (ne^w)^{(1-\theta)/\theta} [s - nF]^{(\theta-1)/\theta}. \quad (\text{A.3})$$

Similarly, equation (14) can be written as:

$$[n\omega^{\theta-1} + (1-n)]^{-1} = [(1-n)e^w]^{(1-\theta)/\theta} [s^* - (1-n)F^*]^{(\theta-1)/\theta}. \quad (\text{A.4})$$

Substituting (A.3) and (A.4) into (A.2) yields:

$$\begin{aligned} & n^{(1-\theta)/\theta} (s - nF)^{(\theta-1)/\theta} - (\theta-1)n^{1/\theta} (s - nF)^{-1/\theta} F \\ & = (1-n)^{(1-\theta)/\theta} (s^* - (1-n)F^*)^{(\theta-1)/\theta} - (\theta-1)(1-n)^{1/\theta} (s^* - (1-n)F^*)^{-1/\theta} F^*. \end{aligned} \quad (\text{A.5})$$

(A.5) is equivalent to (16).